

THEORY OF THE DISINTEGRATION OF A HEATED SURFACE
LAYER, ALLOWING FOR ITS SEPARATION
INTO INDIVIDUAL PHASES

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When the surface layer of a condensed substance is rapidly heated by a laser radiation, a peculiar effect takes place. After the arrival of waves of rarefaction in the central part of the heated layer (from its boundary with the vacuum and from the colder layers in which no energy is dissipated and the pressure is low) the material disintegrates, the surface layer splintering in the opposite direction to the radiation flux (even for concentrations of evolved energy f much smaller than the heat of vaporization Q). For values of f comparable with Q , both evaporated and partially evaporated material flies off. This disintegration is accompanied by the appearance of a reactive force and the development of a mechanical "recoil" momentum.

Experiments confirming the existence of the effect in question were described earlier [1]. "Semi-transparent substances" (colored ice and paraffin and also copper crystal hydrate) were used. The absorption coefficient and the mass of the heated layer underwent no severe changes in the course of heating. The ranges l_0 of the radiation lay between 10^{-2} and $3 \cdot 10^{-2}$ cm. As the characteristic gas-dynamic time in this problem, we may take the time required for sonic perturbations to propagate through the heated layer $t_0 = l_0/c_0$, where c_0 is the velocity of sound in the cold material. In the experiments here described, the sound perturbations were unable to propagate through the heated layer ($t_q \lesssim t_0$) during the period of operation of the pulse t_q (20 nsec), and the energy evolution could be regarded as instantaneous. An estimate of the pressure and the disintegration velocities was given elsewhere [2], and a general picture of the development of splintering and of the (recoil) momentum was presented for $f \ll Q$.

In the Kalmykov experiments [1], substances with a low heat of vaporization were used (the values for the three substances indicated above were 2.5, 0.8, and 0.6 kJ/g, respectively). On increasing f , analogous experiments might be carried out with other materials.

The energy concentration may be increased both as a result of the energy supplied to the surface and also as a result of a reduction in the thickness of the layer. In the latter case sound perturbations are able to propagate through the layer well before the end of energy evolution, and the heating process cannot be regarded as instantaneous.

A model solution of the disintegration of a fixed mass of material heated continuously for a long time was presented in [3]. The material was regarded as a gas with a constant adiabatic index γ , which is only permissible for $f \gg Q$. In the intermediate ranges ($t_0 \gtrsim t_q$ and $t_q \gg t_0$, but $f \lesssim Q$) any estimation of the parameters of the disintegrating material is very difficult, and it is natural to proceed to a numerical solution.

The solution of this problem is not only of interest from the point of view of obtaining a theoretical estimate of the parameters for the conditions of the experiments in question. Frequently the problem of the evaporation of a surface layer of material is considered on the basis of the concept of an "evaporation wave." This is convenient when the evaporation takes place in a narrow (by comparison with the dimensions of the whole problem) layer; either physical detachment occurs (surface evaporation), or else there is a thin transitional layer in which the vapor concentration increases gradually (volume evaporation). The total mass of

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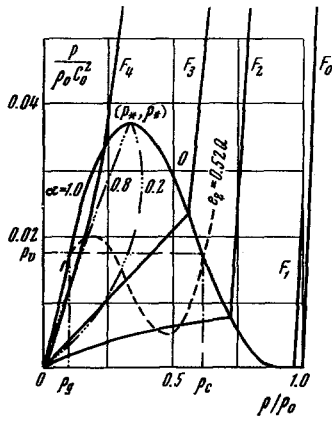


Fig. 1

evaporated material may considerably exceed the mass of the transitional layer (this is associated with an increase in the transparency of the material, for example, a metal, as evaporation proceeds). However, we may also encounter the case in which the whole mass of the evaporated material is nearly equal to the total mass of the layer in which energy evolution is taking place (for example, the absorption coefficient remains constant during the evaporation). The foregoing experiments in fact took place under these conditions. In the case of the fairly hard ultraviolet radiation emitted by exploding sources [4], the absorption coefficient also changes very little until the material is not only evaporated but also ionized. Up to this instant, the mass of the heated layer increases little. Finally, even when optical radiation acts upon metals, there may be cases in which the pressure is high compared with the critical (van der Waals) pressure p_* or even exceeds it, and hence the introduction of a moving boundary in which the phase transformation takes place is quite impossible.

In the case of the heating of a substance by an electric-current pulse (derived from the discharge of a powerful condenser battery or by the penetration of a high-intensity magnetic field) or a powerful flux of fast electrons (in the electron-beam machining of materials), analogous situations arise as the surface layer of the condensed material undergoes intensive heating.

In all these cases, it is natural to have recourse to a direct numerical calculation of the motion of the substance in question, allowing for its possible two-phase nature, i.e., by using an equation of state describing both the condensed material and its gaseous form.

1. The system of equations of gas dynamics (momentum, continuity, and energy) describing plane nonstationary motions of matter in Lagrangian coordinates takes the form

$$\frac{\partial u}{\partial t} + \frac{\partial p}{\partial m} = 0, \quad \frac{\partial v}{\partial t} - \frac{\partial u}{\partial m} = 0, \quad \frac{\partial (e + u^2/2)}{\partial t} + \frac{\partial (pu)}{\partial m} = \frac{\partial f}{\partial t} \quad (1.1)$$

Here u is the mass velocity, p the pressure, v the specific volume, e the internal energy, f the energy evolution in unit mass, $\partial f / \partial t$ the intensity of energy evolution per unit time, t the time, and m the mass coordinate.

The equation of state may be written in the following form:

$$\begin{aligned} e &= e_1 + e_2 = Bv_0\psi(y) + e_2, \quad B = \rho_0 c_0^2, \quad v_0 = \frac{1}{\rho_0} \\ p &= p_1 + p_2 = B\varphi(y) + e_2\rho(\gamma - 1), \\ \gamma &= \gamma(e_2/Bv_0, y), \quad d\psi = -\frac{\varphi}{y^2} dy, \quad y = \frac{\rho}{\rho_0} \end{aligned} \quad (1.2)$$

Here B is the bulk compression modulus, ρ_0 , c_0 , v_0 are the density, the velocity of sound, and the specific volume in the "normal" state, e_1 , p_1 are the "cold" (or "elastic") components of internal energy and pressure, e_2 , p_2 are the hot components, γ is the effective adiabatic index. For the calculations described in the present treatment we took

$$\varphi = y^n - y^k \quad (n \geq k \geq 1) \quad (1.3)$$

The heat of vaporization is then related to Bv_0 in the following manner:

$$Q = Bv_0 (n - k) / [(n - 1)(k - 1)] \quad (1.4)$$

In addition to this, we consider that $\gamma = \gamma(e_2/Bv_0)$, i.e., depends solely on the ratio of the thermal energy to the heat of vaporization. We note that for $\gamma = \text{const}$ the thermal part of the pressure may also be expressed in the form

$$p_2 = e_2\rho(\gamma - 1) = y^\gamma F(s) \quad (1.5)$$

where $F(s)$ is the entropy function.

We assume that the separation of the expanding material into individual phases (the condensed medium and the vapor) follows instantaneously after the change in the thermodynamic parameters, for example, on relaxation after the passage of a shock wave. The boundary of the two-phase region $p = p_V(v)$ is found from the ordinary condition, namely, as the line connecting points satisfying the condition (for $e_2 = \text{const}$)

$$\int_{v_c}^{v_g} p dv = p_v(v_g - v_c) \quad \left(v_g = \frac{1}{\rho_g}, v_c = \frac{1}{\rho_c} \right) \quad (1.6)$$

Here v_g and v_c are the specific volumes of the gaseous (index g) and condensed (index c) phases for a phase-transformation pressure of p_V (Fig. 1). The critical (in the van der Waals sense) point (p_*, ρ_*) is found from the usual relation

$$dp/d\rho = 0, \quad d^2p/d\rho^2 = 0 \quad \text{for} \quad e_2 = \text{const} \quad (1.7)$$

For $n=3, k=2, p_* = 1/27 B, \rho_* = 1/3\rho_0$. At this point $e = 10/9Q$, with $e_1 = 4/9Q$ and $e_2 = 2/3Q$. We note that all these values are independent of γ . For a pressure $p = 0.1p_*$ and $\gamma = 2$, at the boundary of the two-phase region we have $e_c = 0.40Q$, and for $p = 0.01p_*$ we have $e_c = 0.22Q$.

If the pressure calculated from Eq. (1.2) is less than $p_V(\rho)$, the material lies in the two-phase region.

The specific volume and internal energy in this region are described in the following manner:

$$\begin{aligned} v &= v_c(1 - \alpha) + v_g\alpha \\ e &= e_c(1 - \alpha) + e_g\alpha \end{aligned} \quad (1.8)$$

where α is the mass proportion of material converted from the condensed to the gaseous state. Each of the quantities e_c and e_g depends only on the temperature, the following conditions being satisfied:

$$T = T_c = T_g = T_v(p) \quad (1.9)$$

where T_V is the equilibrium temperature of the phase transformation.

Let us consider the question of the adiabatic index γ for the two-phase region. For comparatively low pressures ($p \ll p_*$) the specific volume $v_c \ll v_g$. Hence even for comparatively low vapor concentrations we may neglect the first term on the right-hand side of (1.8) and put $v = v_g\alpha$. Then the expression for e_g may be written as

$$e_g = e - e_c = (e_g - e_c)\alpha = (e_g - e_c)\frac{v}{v_g} = (e_g - e_c)\frac{pv A_w}{RT} \quad (1.10)$$

Here R is the universal gas constant, A_w is the atomic weight; for the vapor phase we have used the equation of an ideal gas. Equation (1.10) may be given the usual form

$$e_g = e - e_c = \frac{pv}{\gamma - 1}, \quad \gamma - 1 = \frac{RT}{A_w Q} \quad (1.11)$$

A long way from the critical point ($p \ll p_*$), Eq. (1.9) is such that the temperature T_V changes very little for a considerable change in p , and $e_g - e_c = Q$. Hence the motion of the two-phase mixture of material occurs practically in an isothermal manner, and to a first approximation we may regard T, e_g, e_c , and γ as constants.

For pressures of $10^2 - 10^3$ bars the average value of $RT_V/A_w Q$ usually equals $1/5$, and hence $\gamma = 1.2$.

Subsequently we shall consider that the internal energy of the vapor $e_g \rightarrow 0$ as $T \rightarrow 0$, while the internal energy of the condensed phase e_c equals Q at $T = 0$.

However, the origin for reckoning the value of e is not very important. For problems relating to the heating and disintegration of a two-phase medium, taking $T_V = \text{const}$, we may therefore put $e_c = 0$ (i.e., we may take as origin the internal energy of the condensed phase at the temperature of the phase transformation), and then Eq. (1.11), the equation of state of the two-phase mixture, will exactly coincide with the equation of state of an ideal gas.

Relaxing isentropics with different $F(s)$ are shown in Fig. 1 (indices 0, 1, 2, 3, 4 correspond to $F/B = 0, 0.027, 0.29, 0.50, \text{ and } 1.28$, respectively). The same figure illustrates the boundary of the two-phase region for the equation of state taken with $n=3, k=2, \gamma=2$ (the grounds for the choice of these values are

given elsewhere [2, 5]). We note that the equation of state used differs little from that of water [6]. In the same Fig. 1 the double dotted and dashed line represents equal concentrations of the gas phase α .

We also considered the motion of the material without allowing for separation into phases, despite the fact that the material already lay within the region $p_V(\rho)$, so that the separation of the kind envisaged was quite feasible. This kind of situation may arise in a very rapid expansion of material in a metastable state. For this case Eqs. (1.2) and (1.3) were everywhere employed without considering (1.4) and (1.6).

2. We carried out our numerical calculation of the equations of gas dynamics by allowing for energy evolution in accordance with a difference scheme based on the method of integral relationships. Since this scheme is clearly of independent interest, we may devote a brief discussion to its underlying principles.

The idea of using integral relationships is not new. This method was described in [7] and used for solving stationary two-dimensional problems in [8] and nonstationary one-dimensional problems involving a shock front in [9]. In these cases approximations relating to the values of the functions at all points of space were used, and only a comparatively small number of integration bands were required, this being due to the smoothness of the solution. The problems now under consideration involve waves of compression and rarefaction interacting with one another, and an approximation extending over the whole range or a considerable proportion of the latter is undesirable, it being better to use interpolation within a small region only. We note that this method is similar to the integro-interpolational method described in [10] and used for parabolic equations. We may also mention that in its initial version (replacing the functions within the cell by their mean values) this approach was used by one of the authors in [11].

Let us consider the equation

$$\frac{\partial M}{\partial t} + \frac{\partial N}{\partial m} + F = 0 \quad (2.1)$$

which for an appropriate choice of M , N , and F represents any of the system (1.1).

Let the whole mass of the problem be divided up (by reference points m_i) into n parts with intervals of δm_i , and let the functions M , N , and F be specified at each point in the j -th layer with respect to time. In each interval (m_{i-1}, m_{i+1}) we approximate the functions M , N , and F by their values at the points m_{i-1} , m_i , m_{i+1} ($i = 1, 2, \dots, n-1$):

$$a_l m^2 + b_l m + c_l = 0 \quad (2.2)$$

Here the symbol l determines which of the three functions M , N , and F the parabola represents. We note that the coefficients a_l , b_l , c_l depend on the values of the l -th function at the points m_i , m_{i-1} , m_{i+1} .

Let us integrate Eq. (2.1) with respect to the intervals

$$\left(\frac{1}{2}(m_i + m_{i-1}), \frac{1}{2}(m_i + m_{i+1})\right) = (m_{i-1/2}, m_{i+1/2}),$$

$i = 1, 2, \dots, n-1$, and with respect to the boundary intervals $(0, m_1/2)$, $((m_{n-1} + m_n)/2, m_n)$.

As a result of this, Eq. (2.1) reduces to a system of ordinary differential equations of the following form:

$$A \frac{dM}{dt} = \Phi \quad (2.3)$$

where A is a three-diagonal matrix of dimensions $(n+1)$

$$\frac{dM}{dt} = \left(\frac{dM_0}{dt}, \dots, \frac{dM_n}{dt} \right), \quad \Phi = (\Phi_0, \Phi_1, \dots, \Phi_n)$$

Then Eq. (2.3) is solved relative to the derivatives dM_i/dt using the well known reduction method for matrices of type A , and (2.3) is converted to the form

$$\frac{dM}{dt} = \Phi' \quad (\Phi' = A^{-1}\Phi) \quad (2.4)$$

The nondegeneracy of the matrix A is quite easily verified.

The resultant system (2.4) is solved by the numerical Euler method with conversion. The time step is chosen from considerations of stability. The stability condition, after verification on model equations of the

$$\frac{\partial u}{\partial t} = K \frac{\partial u}{\partial m} \quad (2.5)$$

type, coincided with the ordinary condition for the equations of gas dynamics:

$$\Delta t^j \leq \min_i (\delta m_i / \rho_i c_i)$$

where ρ_i and c_i are respectively the density and velocity of sound at the point m_i in the j -th layer with respect to time, while Δt^j is the time step from the j -th to the $(j+1)$ th layer.

Let us consider the model equation (2.5) and present some more detailed calculations for the case $\delta m_j = \text{const} = \delta m$, $K \equiv 1$. Under these conditions the coefficients of the parabola (2.2) will take the form

$$a = \frac{u_{i+1} + u_{i-1} - 2u_i}{2\delta m^2}, \quad b = \frac{u_{i+1} - u_{i-1}}{2\delta m}, \quad c = u_i \quad (2.6)$$

(it is assumed that the origin of coordinates is displaced to the point m_i , but this in no way vitiates the generality). Then after integration of Eq. (2.5) we obtain

$$\frac{11}{12} \frac{du_i}{dt} + \frac{1}{24} \left(\frac{du_{i-1}}{dt} + \frac{du_{i+1}}{dt} \right) = \frac{u_{i+1} - u_{i-1}}{2\delta m} \quad (2.7)$$

Similar operations are carried out for all the points i , $i = 1, 2, \dots, n-1$.

After integration with respect to the intervals $(0, m_1/2)$ and $(m_{n-1} + \delta m/2, m_n)$ we obtain two further equations:

$$\begin{aligned} \frac{2}{3} \frac{du_0}{dt} + \frac{5}{12} \frac{du_1}{dt} - \frac{1}{12} \frac{du_2}{dt} &= \frac{6u_1 - 5u_0 - u_2}{4\delta m} \\ \frac{2}{3} \frac{du_n}{dt} + \frac{5}{12} \frac{du_{n-1}}{dt} - \frac{1}{12} \frac{du_{n-2}}{dt} &= \frac{5u_n + u_{n-1} - 6u_{n-2}}{4\delta m} \end{aligned} \quad (2.8)$$

In this way we have obtained an algebraic system of equations in the derivatives du_i/dt ($i = 0, 1, \dots, n$) having dimensions $(n+1)$ and a matrix A , which may readily be seen from Eqs. (2.7) and (2.8).

Remembering that the right-hand sides of (2.7) and (2.8) are taken in the j -th layer with respect to time, in which they are known quantities, we solve this system of equations by the reduction method and derive the system of equations

$$du_i/dt = \Phi_i' \quad (i = 0, 1, 2, \dots, n) \quad (2.9)$$

Applying the numerical Euler method with conversion to (2.9), we obtain at the first stage

$$U_i^{j+1} = u_i^{j-1} + (\Delta t^{j+1} + \Delta t^j) \Phi_i' \quad (2.10)$$

Here from stability considerations a three-layer scheme was chosen.

After the calculation of all the U_i ($i = 0, 1, \dots, n$) on the right-hand sides of the system (2.7), (2.8) all the u_i are replaced by U_i , and this system is again solved for the derivatives. We obtain a new system of equations

$$du_i/dt = \Psi_i' \quad (2.11)$$

Then we solve the following system:

$$du_j/dt = (\Phi_i' + \Psi_i')/2 \quad (2.12)$$

according to the two-layer scheme

$$u_i^{j+1} = u_i^j + \Delta t^j (\Phi_i' + \Psi_i')/2 \quad (2.13)$$

The resultant values of u_i^{j+1} ($i = 0, 1, \dots, n$) are regarded as the final values for the $(j+1)$ th layer. The generalization to the case of system (1.1) is obvious.

We note, finally, that in order to be able to solve problems involving discontinuities in their solutions the Neumann artificial viscosity [12] has to be introduced into the equations of system (1.1).

3. We shall consider that the energy-evolution function for unit mass of material varies in a similar way:

$$\frac{\partial f}{\partial t} = \frac{E_q}{m_0 t_q} \xi \left(\frac{t}{t_q} \right) \eta \left(\frac{m}{m_0} \right) \quad (3.1)$$

Here t_q is the time of action of the source, m_0 is the characteristic mass, E_q is the total energy supplied to unit surface ($E_q/t_q = q$ is the average energy-flux density). This is satisfied, for example, for the case in which radiation falls on the material at a constant flux density q and the absorption coefficient κ is constant:

$$\partial f / \partial t = \kappa q \exp(-\kappa m) \quad (3.2)$$

It was assumed in Section 1 that the equation of state satisfied the law of corresponding states, i.e., the forms of the functions φ , ψ , and γ were unchanged. In this case we may argue on the basis of the similarity of hydrodynamic motions with heating.

Let us introduce some dimensionless variables by referring the dimensional variables to the corresponding characteristic dimensional parameters. For the problems under consideration it is natural to choose as the characteristic value of the pressure p the bulk compression modulus B and as the characteristic of density the initial density ρ_0 . As characteristic of specific internal energy, we may choose Bv_0 (this combination is proportional to the heat of vaporization Q , and for $n=3$ and $k=2$ it equals $2Q$).

Then the equation of state for the dimensionless variables remains in the same form as in Section 1 but with $B=1$, $v_0=1$.

As characteristic velocity it is reasonable to take $\sqrt{Bv_0}$, the "cold" velocity of sound c_0 at normal density, and as characteristic mass the quantity m_0 in the energy-evolution law (3.1). For the case of (3.2) it is natural to put $m_0 = 1/\kappa$. As characteristic time we take the time for sound perturbations to propagate through the layer $t_0 = x_0/c_0 = m_0\rho_0/\sqrt{Bv_0}$. Then the system of equations (1.1) takes the same form for the dimensionless variables as for the dimensional variables, except for the energy equation:

$$\frac{\partial (e + 1/2u^2)}{\partial t} + \frac{\partial (pu)}{\partial m} = E\eta\xi, \quad E = \frac{E_q}{m_0 B v_0} \quad (3.3)$$

Here the dimensionless variables are denoted in the same way as the dimensional. The dimensionless similarity criterion E characterizes the extent to which the energy concentration at the edge of the material exceeds twice its heat of vaporization. In the initial state we shall regard the material as motionless, absolutely cold, and having a normal density. In dimensionless variables the initial data take the form

$$u = e = 0, \quad v = 1, \quad 0 \leq m \leq M, \quad t = 0 \quad (3.4)$$

Here M is the total dimensionless mass of the problem (the dimensional mass equals Mm_0).

The boundary conditions at the "left-hand edge" (from whence the radiation arrives) correspond to an outflow into vacuum ($p=0$), and at the "right-hand edge" an analogous condition or a "rigid" stationary wall:

$$u = 0, \quad m = M \quad (3.5)$$

The layer of heated and disintegrating material is often very thin (by comparison with the whole thickness of the material). It is inconvenient to carry out gas-dynamic calculations in the range $m \gg 1$, since it is clear that in this region a weak shock wave or in an extreme case a wave of moderate intensity will propagate from the zone of energy evolution under the influence of the pressure on a certain surface (the motion of the wave may be described in the quasi-acoustic approximation [13]). This pressure, however, is not prespecified but has to be found from the solution of the gas-dynamic equation with a "soft wall" condition, constituting the relation between the pressure and the velocity in weak waves and waves of moderate intensity:

$$p = (1 + \beta u) u \quad (3.6)$$

where β is a constant close to 1.5. For $p \ll 1$ we have $u \ll 1$, i.e., for weak waves or pressures much smaller than B condition (3.6) transforms into (3.5).

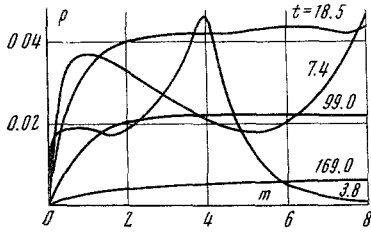


Fig. 2

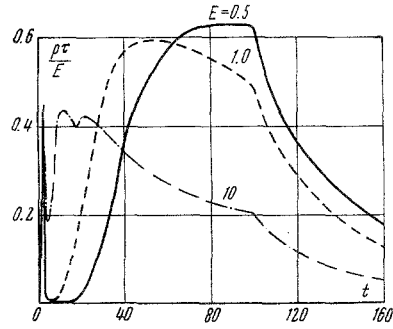


Fig. 3

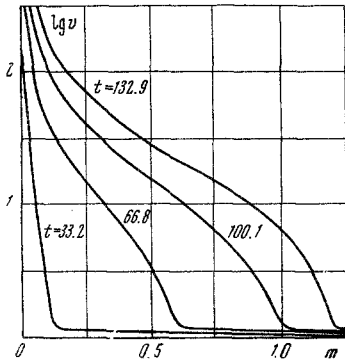


Fig. 4

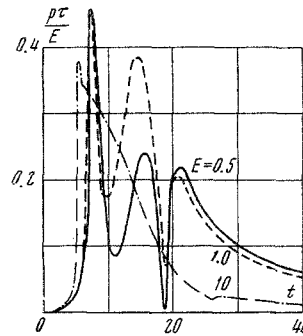


Fig. 5

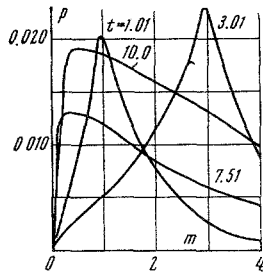


Fig. 6

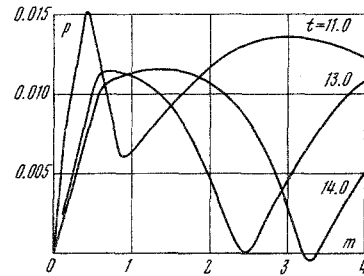


Fig. 7

Thus, in this problem, with exactly the same form of the functions ψ , φ , ξ , η , and γ , we have three parameters: E , M , and the total dimensionless heating time $\tau = t_q/t_0$.

If there is a layer of considerable extent with no energy evolution behind the heated layer, we may use condition (3.6) at an arbitrary (but fairly remote) point $M \gg 1$ (so that the result may be independent of M and β) and consider that the calculation corresponds to $M = \infty$, while the solution depends solely on the two parameters τ and E .

4. Let us give the results of a number of calculations (for $n=3$, $k=2$, $\gamma=2$, and $\beta=1.5$) which we carried out in relation to E and τ . As function f we chose that represented by Eq. (3.2). Then

$$\xi = 1, 0 \leq t \leq \tau; \xi = 0, t > \tau; \eta = \exp(-m) \quad (4.1)$$

The parameter τ shows how many times during the heating period (i.e., in a time t_q) sound perturbations would be able to penetrate through a layer of thickness l_0 (with the initial velocity of sound c_0), i.e., it characterizes the duration of the heating process. Here we describe the disintegration process taking place in the material for $10 \geq E \geq 0.5$, $100 \geq \tau \geq 10$.

Before giving the results of the calculations of the "dimensionless" series, we would mention that these may be used for finding the pattern of the motion and determining all the parameters for various "di-

dimensional" versions. For example, we may indicate that, for a heat of vaporization $Q=1$ kJ/g and an initial density of the material $\rho_0=1$ g/cm³, according to the equation of state used, $B=20$ kbars, $p_*=0.7$ kbar, and $c_0=1.4$ km/sec. Approximately the same values Q , ρ_0 , B , and c_0 applied to the materials used in the experiments [1]. In [1] the thickness of the layers was of the order of 10^{-2} cm. Here we consider thinner layers (or greater interaction times). Let $l_0=10^{-3}$ and 10^{-4} cm; then the time t_0 for the propagation of sound through the layer equals 7 and 0.7 nsec. For a dimensionless duration of the process $\tau=10$ and 100 we obtain $t_q=70$ nsec, which corresponds to a typical duration of the process of energy supply in a "giant" pulse.

The characteristic mass m_0 of the layer equals 10^{-3} and 10^{-4} g/cm², so that $m_0 2Q=2$ and 0.2 kJ/g, and the dimensionless parameter $E=0.5-10$ corresponds to values of $E_q=1-20$ J/cm² for $\tau=10$ and $0.1-2$ J/cm² for $\tau=100$ (or flux densities $q=20-400$ and $2-40$ MW/cm², respectively). A laser pulse may be extended, let us say, up to 500 nsec. Then, for the same values of the dimensionless parameters, in order to create the same pressure, velocity, and gas density (but over a 10-times greater time) as in the case of a pulse 50 nsec long, we require that the initial thicknesses of the heated layers should be increased to 10^{-2} and 10^{-3} cm, the energies by a factor of 10, while the flux density of the radiation should remain the same.

When optical radiation acts upon metals, the characteristic thickness l_0 of the heated layer is smaller, of the order of 10^{-5} cm. Let $Q=1$ kJ/g, $\rho_0=10$ g/cm³, $B=200$ kbars, $p_*=8$ kbars, $c_0=1.4$ km/sec (parameters similar to those of lead). Then we obtain $t_0=0.07$ nsec. For $\tau=10$ we have $t_0=0.7$ nsec. For $m_0=10^{-4}$ g/cm² we obtain $m_0 2Q=0.2$ J/cm², and the values of the parameter $E=0.5-10$ correspond to $E_q=0.1-2$ J/cm² and $q=140-2800$ MW/cm².

Usually when considering the effect of laser radiation on metals we allow for two additional factors: the thermal conductivity and the change taking place in the transparency of the metals during disintegration. Let the thermal diffusivity be 0.1 cm²/sec (the value for molten lead at $T=1000^\circ\text{K}$; for higher temperatures no data are available). Then in a time $t=10^{-9}$ sec the expansion of the heated layer, allowing for thermal conductivity, is approximately 10^{-5} cm, i.e., the pattern of the process is not very seriously distorted.

Thus the results of our calculations in the range of q under consideration may be used to give an approximate description of the initial stage in the action of a laser beam on metals for times over which the expansion is negligible.

Let us consider the case $\tau=100$, $E=10$. (Subsequently in this Section we shall use dimensionless variables.) Figure 2 illustrates the pressure distribution with respect to mass at various instants of time. We see how a pressure wave passing into the inside of the material is formed. During almost the whole time of energy supply the pressure is close to the van der Waals critical value ($p_*=0.037$).

The pressure relation $p(t)$ at a depth $m=2$ is shown for the same case in Fig. 3 (by the dotted and dashed line); $p(t)$ is referred to the E/τ dimensionless flux density. For comparison, the same figure shows analogous relationships for $E=1$ (broken line) and for $E=0.5$ (continuous line) at the same depth and for the same duration τ . In all cases a sharp pressure peak is clearly to be seen.

Figure 4 shows the distribution of $v(m)$ at different instants of time t for the case $\tau=100$, $E=0.5$.

We see that the density of the heated layer falls considerably in comparison with the initial density (by several orders of magnitude). It should be noted that it is not only the layer in which the main part of the energy is liberated which disintegrates. Thus at the instant $t=100$ a considerable fall starts taking place in the density of the layer with mass $m \leq 1$, in which the concentration of energy evolved equals $0.37Q$, and at the moment $t=133$ the density in the layer with $m \leq 1.2$, in which the concentration of energy evolved equals $0.30Q$, also starts falling sharply. Since these layers expand with small velocities, a characteristic "spur" is created, and we may arbitrarily speak of a boundary between the evaporated and nonevaporated layers.

For large time values, deeper layers start disintegrating. This is associated with the fact that in the equation of state taken for the present series of cases the curve of phase equilibrium proceeds in such a manner that the temperature of the phase transformation $T_v(p) \rightarrow 0$ as $p \rightarrow 0$. For such an equation of state all the material, however much heated, ultimately disintegrates. In actual practice, however, for low phase-transformation temperatures the process of volume vaporization can hardly occur at all. We note that if we consider the pattern of the process for any finite time comparable with the period of action, then as a result of the low velocity of disintegration of these layers they will hardly be able to expand at all. For $\tau=100$

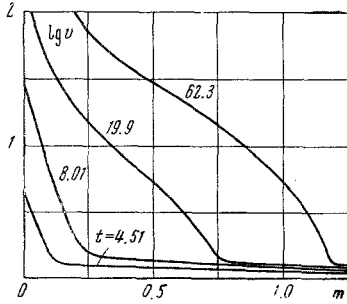


Fig. 8

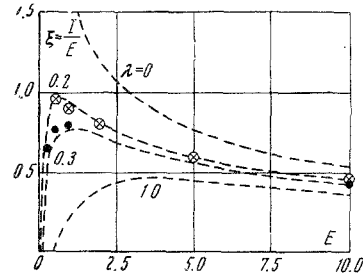


Fig. 9

and 10 our calculation extended to $t = 140$ and 60 ; in calculating the momentum, the pressure drop was subsequently regarded as taking place in accordance with a power law.

Let us now consider the case $\tau = 10$, $E = 0.5$.

In Fig. 5 the continuous line indicates the change in pressure with time at a depth $m = M = 8$. For comparison, the same figure shows analogous relationships for the cases $\tau = 10$, $E = 1$ (broken line) and $\tau = 10$, $E = 10$ (dotted and dashed line) at the same depth (pressures divided by E/τ). We notice the sharp fall in pressure at the instants $t = 18-20$ for $E = 0.5$ and 1.0 . On further reducing E , the pressure becomes negative, and splintering may occur. This may clearly be seen in Figs. 6 and 7, in which the pressure is shown in relation to mass at various instants of time with $E = 0.5$. We see that the initial perturbation with an exponential leading edge moves into the deeper layers of the material (at the moments of time $t \approx 1-4$). Then the pressure rises again as a result of the continuing energy evolution (at the end of energy evolution $t = \tau = 10$). After the conclusion of energy evolution ($t > \tau$) there is a sharp fall in pressure in the region $m \approx 1$, propagating into the deeper layers. We even observe negative pressures (in this region the separation of the material into phases was not taken into account), but in this version they are small in magnitude. This is evidently associated with the fact that in certain layers which have not yet evaporated the store of thermal energy is still quite large. The expansion of this layer is not particularly great, as it is compensated by the back pressure arising from the action of the escaping vapor. After the end of energy evolution and the corresponding reduction in vapor pressure, a sharp expansion occurs in the direction of the vacuum, accompanied by an attraction acting on the adjacent cold layers and dragging them in the same direction.

This is supported by the velocity-distribution graph, which is not reproduced here. At instants of time close to $t = 10$ we observe sharp changes in the velocity of the layers adjacent to the evaporating layer. First these layers acquire negative velocities (motion in the direction of the vacuum), then a reverse motion occurs, as the elastic forces prevent their detachment. As the pressure falls, so does the phase-transformation temperature, and the evaporation of several additional layers becomes possible.

Figure 8 shows the distribution of specific volume for $E = 10$, $\tau = 10$. We see that at the instant at which energy evolution ceases, material with a mass of $m \approx 0.80$ flies apart, the energy concentration for this being approximately $0.45 Q$, while at the instant $t = 62$ the same occurs for the layer with mass $m = 1.2$, where $f = 0.30 Q$. We note that in this case, at no point of the material does the energy concentration exceed the heat of vaporization, even at the end of energy evolution; nevertheless, disintegration of the partially evaporated material takes place. As in the other cases a narrow zone of sharp density reduction appears.

With increasing period of energy evolution the dynamic effects play a less and less significant part. The partially-evaporated material disintegrates with very low velocities since a considerable proportion of the energy has to be spent in evaporation. The negative stresses vanish completely; splintering becomes quite impossible.

5. By comparing the $p(t)$ relationships for various values of E and τ , we see that the rise in pressure at the first maximum is in approximate agreement with the value estimated in the following manner (here and subsequently we return to dimensional variables). Let the material be stationary. Then, as a result of the energy evolution (uniform with respect to mass) in it, the pressure increases in accordance with the law

$$p = (\gamma - 1) Et / (\rho_0 m_0 t_0) = (\gamma - 1) Et / (x_0 t_0) \quad (5.1)$$

(pressure of completely "thermal" origin).

The wave of rarefaction propagates through a uniformly heated layer of mass $\frac{1}{2}m_0$ (to the center of the heated layer) in a time $\frac{1}{2}t_0$. In this time the pressure reaches the value

$$p = E (\gamma - 1) / (2t_2 \epsilon_0) \quad (5.2)$$

After the waves of rarefaction have propagated through the heated layer, the pressure in the latter should fall. This pressure may be determined on the basis of a thermoelastic solution of the linearized system derived from (1.1) for $\Delta\rho/\rho_0 = \Delta y \ll 1$. Here we simply present estimates.

The main part of the energy is consumed in heating the substance. Hence in the range $0 \leq m \leq m_0$

$$e = e_2 = \kappa qt \quad (5.3)$$

The increasing thermal pressure is compensated by the negative elastic pressure

$$p = B \Delta y + e_2 \rho_0 (\gamma - 1) \approx 0 \quad (5.4)$$

Hence

$$\Delta y = - (\gamma - 1) \rho_0 \kappa qt / B \quad (5.5)$$

In addition to this, using (5.5), we obtain the following from the equation of continuity:

$$u = (\gamma - 1) q / B \quad (5.6)$$

From the momentum equation we have

$$p = um / t = (\gamma - 1) q / (B \kappa t) = p^\circ (t^\circ / t) \quad (5.7)$$

where p° is the pressure at the instant of time t° determined from (5.1). Naturally this pressure only acts at the boundary of the heated layer with the cold layers, while at the boundary $m=0$, as before, $p=0$. When evaporation begins, the pressure again starts rising, and then after a considerable expansion of the layer into the vacuum it falls, roughly speaking, in accordance with the "gas" law [3]

$$p = \chi \left(\frac{3(\gamma-1)q}{(3\gamma-1)\kappa\pi t} \right)^{1/2} \quad (5.8)$$

where χ is a coefficient allowing for the redistribution in the energy with respect to mass (for the case of uniform prolonged heating with an exponentially falling law of energy evolution $\chi \approx 0.8$).

The latter result is not obvious for a two-phase mixture in which heat is consumed in evaporation. The result may be explained by the fact that at low pressures the adiabatic index of such a mixture may be regarded as approximately constant, as indicated in Section 1.

6. Let us now consider the dependence of the momentum I on the energy concentration and duration. Figure 9 shows the ratio $\xi = I\sqrt{Bv_0}/E_Q$ for various E and τ obtained by numerical calculation (the crosses are for $\tau = 10$, and the points for $\tau = 100$). We see that an increase in the period of heating leads to a certain reduction in momentum, but in general the $\xi(\tau)$ relationship is quite weak. This is quite natural for the range $f \gg Q$ (or $E \gg 1$), since in this range we have a limiting mode of escape of the heated gas [3] corresponding to $\tau \rightarrow \infty$ (relation (5.8) was in fact derived for this condition). The gas parameters for $\tau \gg 1$ are similar to the parameters in the limiting mode and depend very little on τ .

The value of I in the case of $E_Q/(m_0 Q) \gg 1$ may be approximately estimated from the expression

$$I = \chi \int_0^{m_s} \sqrt{2(f(m) - \lambda Q)} dm \quad (6.1)$$

where λ is the average proportion of the internal energy expended in evaporation, while m_s is determined from the relation $f(m_s) = \lambda Q$ (m_s is the arbitrary or nominal limit of evaporation). The quantity λ is an unknown parameter. It is simply clear that for $E \gg 1$ the value of λ is not very great.

The values of ξ obtained from Eq. (6.1) are shown in Fig. 9 by the broken line for $\lambda = 0, 0.2, 0.3$, and 1.0. We see that the numerical calculations described above agree fairly well with the values of $\lambda = 0.2$ and 0.3. This may be explained in the following way. For the material to start evaporating at pressures below critical (in a three-dimensional and quasi-equilibrium fashion), an energy approximately equal to $3R_V T(p)$ must be expended, where T_V is the temperature of the phase transformation at medium pressures during

the evolution of the energy. A typical value of $RT_v/A_w Q \approx 0.1-0.2$, and hence the enthalpy h corresponding to the onset of evaporation is $0.3-0.6 Q$.

For short times ($\tau \ll 1$) the creation of momentum is even possible for $E \ll 1$ (in contrast to the cases here considered $\tau \gg 1$). For the material to disintegrate, it is actually not essential that it should evaporate completely. It is sufficient, by expending a comparatively small amount of energy, to "break it up" within a certain layer. This kind of rupture is quite possible in the case of instantaneous heating, since after the disconnection of the source the interacting relaxation waves propagating from the boundaries of the heated material lead to the development of negative stresses.

However, as we have already indicated, such "dynamic" effects are observed for not instantaneous but fairly prolonged heating. The effects may clearly be seen in Fig. 7 (in the case of $\tau = 10$, $E = 0.5$), while in cases with still lower E values splintering should occur (these effects have not been considered in the present treatment). It would appear that as the duration falls and the pressure rises, there should be an increase in the typical values of T_v and hence $h(T_v)$ and λ . Clearly the increase in the part played by dynamic effects will lead to a reduction in λ with diminishing τ . Such an interpretation of the results of the numerical calculations, however, must by no means be regarded as the final one.

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